

Advanced Macroeconomics II

Assignment 3

(Submission Time: 5:00 pm, 2 July 2007)

1. In the OLG model discussed in class, assume that the utility function of an individual born at time t is given by:

$$W_t = u(c_{1t}, c_{2t+1}) = (c_{1t})^\alpha + \beta (c_{2t+1})^\alpha, \quad \alpha, \beta \in (0, 1). \quad (1)$$

Each individual is endowed with one unit of perishable consumption good when young and nothing when old.

(a) What are the values of c_{1t} and c_{2t+1} in the barter equilibrium?

(b) Suppose that at time 0 the government gives to the old H_0 units of money and that the initial old and every generation thereafter believe that they will be able to exchange money for goods. What are the values of c_{1t} and c_{2t+1} in the steady-state monetary equilibrium? Does the introduction of money improve welfare?

(c) Now suppose that the nominal money stock grows at a constant rate σ and that new money is introduced into the economy through interest payments to money holders. Is money superneutral in this case? Explain your answer.

2. Consider the Sidrauski model discussed in class, assume that the representative household's utility function is given by:

$$u(c) = \frac{(c^\beta m^{1-\beta})^{1-\epsilon}}{1-\epsilon}, \quad (2)$$

where $\epsilon > 0$ and $0 < \beta < 1$. The production function is:

$$Y = AK^\alpha N^{1-\alpha} \quad \text{or} \quad y = Ak^\alpha, \quad (3)$$

where $0 < \alpha < 1$ and $A > 0$. Assume that capital does not depreciate. Also assume that the money supply increases at a constant rate σ and that the money supply expands through lump-sum transfers to households.

(a) Find the first-order conditions for the representative household's optimization problems.

(b) Use the first-order conditions in (a) to write down the demand for real balances as a function of consumption c and the nominal interest rate i . How does the demand for real balances depends on consumption c and the nominal interest rate i ?

(c) Set up and solve the optimization problem of the representative firm.

(d) What are the steady-state values of inflation π^* , real money balances m^* , capital per worker k^* and consumption per worker c^* ? Is money superneutral?

(e) What is the optimal steady-state growth rate of money σ^* ? Explain.

3. Consider the Seignorage-Inflation model discussed in class. Suppose that desired real money holdings are given by

$$m(t) = \frac{M(t)}{P(t)} = a - bi(t) + c\bar{Y} = C - b\pi(t), \quad b > 0,$$

where $C \equiv a + c\bar{Y} - b\bar{r}$.

(a) Suppose that the public immediately adjusts its money holdings to changes in the economic environment. What is the maximum feasible amount of seignorage S^* in the steady state?

(b) Consider the economy in the short run. Suppose that the public adjusts its money holdings gradually toward desired holdings according to

$$\frac{\dot{m}(t)}{m(t)} = \beta[m^*(t) - m(t)], \quad 0 < \beta < 1/b. \quad (4)$$

Suppose that the government needs to finance an amount of real purchases G ($> S^*$) by printing money. Analyze the dynamics of the real money stock $m(t)$ and inflation $\pi(t)$.

(c) Suppose that $G < S^*$ in (b). Show that inflation will not degenerate into hyperinflation.

4. Consider the Clarida-Gali-Gertler (1999) model. The model consists of the following two equations:

$$x_t = -\varphi[i_t - E_t\pi_{t+1}] + E_t x_{t+1} + g_t, \quad (5)$$

$$\pi_t = \lambda x_t + \beta E_t \pi_{t+1} + u_t, \quad (6)$$

where

$$g_t = \mu g_{t-1} + \hat{g}_t, \quad \mu \in [0, 1], \quad (7)$$

$$u_t = \rho u_{t-1} + \hat{u}_t, \quad \rho \in [0, 1], \quad (8)$$

The policy objective function is given by:

$$\max -\frac{1}{2} E_t \left\{ \sum_{i=0}^{\infty} \beta^i [\alpha x_{t+i}^2 + \pi_{t+i}^2] \right\}. \quad (9)$$

- (a) Where do the two equations (5) and (6) come from?
- (b) According to (5), current output depends negatively on the real interest rate and positively on expected future output. Explain why.
- (c) According to (6), inflation depends positively on the output gap and expected future inflation. Explain why.
- (d) Show that, under the optimal policy without commitment, the central bank should adjust the nominal interest rate more than one-for-one with expected future inflation.
- (e) Discuss the gains from commitment.