III Economic Growth (continued)

F Endogenous Growth: Aghion-Howitt (1992) Model

1 Introduction

- Source of economic growth: industrial innovations that improve the quality of products.
- Schumpeter's idea of creative destruction: high-quality products render low-quality products obsolete.
- The amount of research this period depends negatively on the expected amount of research next period through the creative destruction effect and the wage effect.
- The paper identifies four effects: the appropriability effect, the intertemporal-spillover effect, the business-stealing effect and the monopoly-distortion effect.
- The laissez faire (decentralized) equilibrium is not optimal: the laissez faire growth rate may be more or less than optimal. Different from other endogenous growth models, the decentralized equilibrium growth rate can be too high!

2 The Model

There are three classes of tradeable objects: labor, a consumption good and an intermediate good.

- Preferences: Constant rate of time preference $r > 0$,

$$U = \mathcal{E}_0 \int_0^\infty e^{-rt} c(\tau) d\tau. \tag{1}$$

- Consumption Good Production: The consumption good is produced using the fixed quantity M of unskilled labor and the intermediate good x, subject to constant returns. The production function is

$$y_t = A_t x_t^\alpha, \quad 0 < \alpha < 1. \tag{2}$$

- Intermediate Good Production: The intermediate good is produced using skilled labor L.

$$x_t = L_t. \tag{3}$$

- Innovations: R&D produces a random sequence of innovations with the Poisson arrival rate λn_t, where n_t is the flow of skilled labor used in R&D and $\lambda > 0$ is a parameter.

  Each innovation discovers a *new* intermediate good whose use increases the productivity parameter A_t by a factor $\gamma > 1$.

$$A_t = A_0 \gamma^t, \quad t = 0, 1, 2, ..., \tag{4}$$

  where A_0 is given. A successful innovator obtains a patent that lasts forever, but the innovator can enjoy monopoly profits until the next innovation.

- Markets: All markets are perfectly competitive except that for intermediate goods.

3 Decentralized Equilibrium

- Consumption Good Producer:

$$\max_{x_t} \{A_t x^\alpha - p_t x_t\}$$

The first-order condition:

$$p_t = \alpha A_t x_t^{\alpha-1}.$$

- Intermediate Good Producer:

$$\pi_t = \max_{x_t} \{p_t x_t - w_t x_t\}$$

$$= \max_{x_t} \{\alpha A_t x_t^\alpha - w_t x_t\}$$

The first-order condition:

$$\alpha^2 A_t x_t^{\alpha-1} = w_t.$$

which implies:

$$x_t = \left(\frac{\omega_t}{\alpha^2}\right)^{\frac{1}{\alpha-1}}$$

$$p_t = \frac{w_t}{\alpha} = \frac{A_t \omega_t}{\alpha}$$

$$\pi_t = \left(\frac{1-\alpha}{\alpha}\right) w_t x_t = A_t \left(\frac{1-\alpha}{\alpha}\right) \omega_t x_t$$

where $\omega_t \equiv w_t / A_t$ is the "productivity-adjusted wage".

- Innovators:

$$\max_{z_t} \lambda z_t V_{t+1} - w_t z_t,$$

where z_t is the amount of skilled labor used and V_{t+1} is the value of the $t+1st$ innovation. The first-order conditions are

$$w_t \geq \lambda V_{t+1}, \quad n_t > 0, \quad \text{with at least one equality,} \tag{5}$$

where n_t is the economy-wide flow of skilled labor used in R&D. The value of innovation is given by

$$V_{t+1} = \frac{\pi_{t+1}}{r + \lambda n_{t+1}}.$$

Note that all R&D is conducted by outside R&D firms rather than by the incumbent monopolist because the value to the monopolist of making the next innovation $(V_{t+1} - V_t)$ is strictly less than the value to an outside firm (V_{t+1}).

- Capital Markets: There are perfect capital markets.

- Perfect Foresight Equilibrium:
  Since $w_t = \alpha^2 A_t x_t^{\alpha-1}$, $\pi_{t+1} = A_{t+1}\left(\frac{1-\alpha}{\alpha}\right)\omega_{t+1}x_{t+1}$ and $N = x_t + n_t$, (5) implies

$$\frac{\alpha^2(N - n_t)^{\alpha-1}}{\lambda} \geq \frac{\gamma\left(\frac{1-\alpha}{\alpha}\right)\alpha^2(N - n_{t+1})^{\alpha}}{r + \lambda n_{t+1}}, \quad n_t \geq 0,$$

with at least one equality. This condition determines n_t as a function of n_{t+1}:

$$n_t = \psi(n_{t+1}). \tag{6}$$

Let $c(n_t)$ and $b(n_{t+1})$ be respectively the "marginal cost of R&D" and "marginal benefit of R&D":

$$c(n_t) \equiv \frac{\alpha^2(N-n_t)^{\alpha-1}}{\lambda}, \quad b(n_{t+1}) \equiv \frac{\gamma\left(\frac{1-\alpha}{\alpha}\right)\alpha^2(N-n_{t+1})^{\alpha}}{r+\lambda n_{t+1}}.$$

An increase in n_{t+1} reduces n_t (a) by raising the future wage rate and thus lowering the flow of profits $\gamma\left(\frac{1-\alpha}{\alpha}\right)\alpha^2(N-n_{t+1})^{\alpha}$ and (b) by raising the rate of creative destruction λn_{t+1}.

- PFE: A perfect foresight equilibrium (PFE) is defined as a sequence $\{N_t\}_0^\infty$ satisfying (6) for all $t \geq 0$. A stationary equilibrium corresponds to a PPE with $n_t = \hat{n}$, where $\hat{n} = \psi(\hat{n})$.

$$\hat{n} = \frac{\lambda\gamma(1-\alpha)N - \alpha r}{\lambda[\alpha + \gamma(1-\alpha)]}.$$

Obviously, $\hat{n} > 0$ if $\lambda\gamma(1-\alpha)N/(\alpha r) > 1$.

- R&D employment in stationary equilibrium: $\hat{n}$ increases with: (a) a decrease in r; (b) an increase in γ; (c) an increase in N; or (d) an increase in λ.

- Minimal monopoly power (Lerner's measure): $(1-\alpha) > (1-\alpha^*)$, where

$$\alpha^* = \frac{\lambda\gamma N}{r + \lambda\gamma N} < 1.$$

- Balanced growth:

$$y_t = A_t F(N - \hat{n})$$

$$y_{t+1} = \gamma y_t, \quad y_{t+1} = \gamma y_t$$

$$\ln y(\tau+1) = \ln y(\tau) + \epsilon(\tau) = \ln y(\tau) + \lambda\hat{n}\ln\gamma + e(\tau)$$

where $e(\tau)$ is iid with $E(e(\tau)) = 0$ and Var $(e(\tau)) = \lambda \hat{n} (\ln \gamma)^2$. The average growth rate and the variance of the growth rate are respectively

$$AGR = \lambda \hat{n} \ln \gamma \quad \text{and} \quad VGR = \lambda \hat{n} (\ln \gamma)^2.$$

4 Welfare Properties of the Stationary Equilibrium

- The social planner chooses n to maximize (1), which is equivalent to

$$U = \int_0^\infty e^{-r\tau} \sum_{t=0}^{\infty} \Pi(t,\tau) A_t F(N-n) d\tau, \tag{7}$$

where $\Pi(t,\tau)$ is the probability that there will be exactly t innovations up to time τ. Since the innovation process is Poisson with parameter λn, we have

$$\Pi(t,\tau) = (\lambda n \tau)^t e^{-\lambda n \tau} / t! \tag{8}$$

Combining (7) and (8) gives

$$U = \frac{A_0 F(N-n)}{r - \lambda n(\gamma - 1)}.$$

The first-order condition:

$$\frac{\alpha (N - n^*)^{\alpha - 1}}{\lambda} = \frac{(\gamma - 1)(N - n^*)^\alpha}{r - \lambda n(\gamma - 1)},$$

leading to

$$n^* = \frac{\lambda(\gamma - 1)N - \alpha r}{\lambda(\gamma - 1)(1 - \alpha)}$$

Comparing this with the(stationary) equilibrium condition

$$\frac{\alpha^2(N-\hat{n})^{\alpha-1}}{\lambda} = \frac{\gamma\left(\frac{1-\alpha}{\alpha}\right)\alpha^2(N-\hat{n})^{\alpha}}{r+\lambda\hat{n}},$$

we can see four differences:

(i) The *intertemporal-spillover effect* (the social discount rate $r - \lambda n(\gamma - 1)$ vs. the private discount rate $r + \lambda n$): The social discount rate is less than the rate of interest, whereas the private rate is greater. This difference reflects the intertemporal spillover effect that the social planner takes into account while the private R&D firm does not.

(ii) The *appropriability effect* (total output $(N - n)^\alpha$ vs. the flow of profits $\alpha(1 - \alpha)(N - n)^\alpha$): Innovators are unable to appropriate all the benefits generated by an innovation.

(iii) The *business-stealing effect* (the factor $(\gamma - 1)$ vs. the factor γ): The social planner takes into account the loss to the previous monopolist caused by an innovation white the private R&D firm does not.

(iv) The *monopoly-distortion effect* (the marginal product $\alpha(N - n)^{\alpha-1}$ vs. the wage $\alpha^2(N - n)^{\alpha-1}$): The social cost of R&D employment is higher than the private cost because in the decentralized equilibrium the alternative user of skilled labor is a monopolist.

- The laissez-faire (decentralized equilibrium) growth rate $>$ ($=, <$) the optimal growth rate: The intertemporal-spillover and appropriability effects tend to make the laissez-faire growth rate less than optimal, whereas the business-stealing and monopoly-distortion effects tend to do the opposite.

When the size of innovation γ is large, $\hat{n} < n^*$. When the size of innovation is not too large and there is much monopoly power (α is small), $\hat{n} > n^*$.