III Economic Growth (Continued)

B The Solow Model

1 Assumptions

- Closed economy
- Household/producer

$$Y(t) = F[K(t), L(t), t]$$

- Homogeneous good
- Constant savings rate

$$s(t) = s,\ 0 < s < 1$$

- Constant capital depreciation ($\delta,\ 0 < \delta < 1$)
- Constant population growth

$$L(t) = e^{nt},\ \ L(0) \equiv 1$$

2 Physical Capital Path

$$\dot{K} = I - \delta K = sF(K, L, t) - \delta K$$

3 The Neoclassical Production Function

- Production function

$$Y = F(K, L)$$

- Properties

  ‡ Positive and diminishing marginal products

  $$\frac{\partial F}{\partial K} > 0, \ \ \frac{\partial F}{\partial L} > 0$$

  $$\frac{\partial^2 F}{\partial K^2} < 0, \ \ \frac{\partial^2 F}{\partial L^2} < 0$$

  ‡ Constant returns to scale

  $$F(\lambda K, \lambda L) = \lambda F(K, L), \ \ \forall \lambda > 0$$

  ‡ Inada conditions

  $$\lim_{K \to 0} F_K = \lim_{L \to 0} F_L = \infty$$

  $$\lim_{K \to \infty} F_K = \lim_{L \to \infty} F_L = 0$$

- Intensive form of production function

  $$y = f(k), \ \ \text{where}\, y \equiv Y/L, k \equiv K/L, f(k) \equiv F(k, 1)$$

  ‡ Properties of $f(k)$

  $$\frac{\partial Y}{\partial K} = f'(k), \ \ \frac{\partial Y}{\partial L} = f(k) - kf'(k)$$

  $$\lim_{k \to 0} f'(k) = \infty, \ \ \lim_{k \to \infty} f'(k) = 0$$

- An example : Cobb-Douglas production function

  $$Y = AK^{\alpha}L^{1-\alpha}, \ \ y = Ak^{\alpha}$$

4 The Fundamental Dynamic Equation for the Capital Stock

$$\frac{\dot{K}}{L} = sf(k) - \delta k$$

$$\dot{k} \equiv \frac{d(K/L)}{dt} = \frac{\dot{K}}{L} - nk$$

$$\Rightarrow$$

$$\dot{k} = sf(k) - (n + \delta)k$$

5 The Steady State

- A steady state – all variables grow at constant rates (Figure 1)

$$\dot{k}/k = 0 \Rightarrow sf(k^*) = (n + \delta)k^* \Rightarrow k^*$$

$$\Rightarrow y^* = f(k^*), \;\; c^* = (1 - s)f(k^*)$$

$$\Rightarrow \gamma_k^* = \gamma_y^* = \gamma_c^* = 0$$

$$\Rightarrow \gamma_K^* = \gamma_Y^* = \gamma_C^* = n$$

- **Conclusion:** The steady state growth rates of *per capita* output, capital and consumption are all independent of changes in the level of technology, the saving rate, the rate of population growth, and the depreciation rate.

6 Transitional Dynamics

- The process of an economy converging to its steady state (Figure 2)

  $$\dot{k} = sf(k) - (n+\delta)k$$

  $$\Rightarrow \quad \gamma_k \equiv \frac{\dot{k}}{k} = \frac{sf(k)}{k} - (n+\delta)$$

  $$\gamma_y \equiv \frac{\dot{y}}{y} = \frac{f'(k)\dot{k}}{f(k)} = [\frac{kf'(k)}{f(k)}]\gamma_k$$

  $$\Rightarrow \quad \gamma_y = sf'(k) - (n+\delta)Sh(k), \;\; Sh(k) \equiv \frac{kf'(k)}{f(k)}$$

  $$\frac{\partial \gamma_y}{\partial k} = \frac{f''(k)k}{f(k)}\gamma_k - \frac{(n+\delta)f'(k)}{f(k)}[1 - Sh(k)]$$

  $\Rightarrow \quad \frac{\partial \gamma_y}{\partial k} < 0$ if (a) $k < k^*$, or (b) k is close to k^* if $k > k^*$.

- **Conclusion:** Less-advanced economies have higher growth rates.

7 Policy Experiments

- A permanent increase in the saving rate (Figure 3)

  ⇒ Temporary positive growth in per capita output

  ⇒ Permanently higher *levels* of per capita capital and output

  ⇒ No change in the long-run growth *rates* of per capita capital and output

8 Absolute and Conditional Convergence

- *Absolute convergence*:

$$\frac{\partial \gamma_k}{\partial k} = s[f'(k) - f(k)/k]/k < 0$$

‡ Note: Empirical studies provide mixed results.

- *Conditional convergence*:

$$\gamma_k = (n+\delta)[\frac{f(k)/k}{f(k^*)/k^*} - 1]$$

‡ Note: Empirical evidence supports the hypothesis of conditional convergence.

9 The Solow Model with Labor-Augmenting Technological Progress

$$\dot{K} = sF[K, LA(t)] - \delta K, \;\; A(t) = A(0)e^{xt}$$

$$\Rightarrow \dot{k} = sF[k, A(t)] - (n+\delta)k$$

- Steady state

$$\gamma_k^* = x$$

$$\gamma_y^* = F[k, A(t)] = kF[1, A(t)/k] = x$$

$$\gamma_c^* = x$$

$$\Rightarrow \gamma_K^* = \gamma_Y^* = \gamma_C^* = x + n$$

‡ **Conclusion:** The steady state growth rates of *per capita* output, capital and consumption grow at the same rate as the exogenous technological progress. They are all independent of changes in the level of technology, the saving rate, the rate of population growth, and the depreciation rate.

- Transitional dynamics

‡ Notations: $\hat{k} \equiv k/A(t),\ \hat{y} \equiv Y/[LA(t)]$

$$\hat{y} = F(\hat{k}, 1) \equiv f(\hat{k})$$

$$\gamma_{\hat{k}} = sf(\hat{k})/\hat{k} - (x + n + \delta)$$

‡ The transitional dynamics are similar to the case where there is no technological progress

10 Dynamic Inefficiency and the Golden Rule

Since $c^* = (1 - s)f(k^*)$ and $sf(k^*) = (\delta + n)k^*$, we have

$$c^*(s) = f(k^*(s)) - (\delta + n)k^*(s)$$

Choosing s to maximize $c^*(s)$ gives the golden rule of capital accumulation

$$f'(k_{gold}) = \delta + n$$

where $k_{gold} = k^*(s_{gold})$. If $s > s_{gold}$, then $c^* < c_{gold}$. That is, the economy is oversaving in the sense that per capita consumption at all points in time can be raised by lowering the saving rate. An economy that oversaves is said to be *dynamically inefficient*.

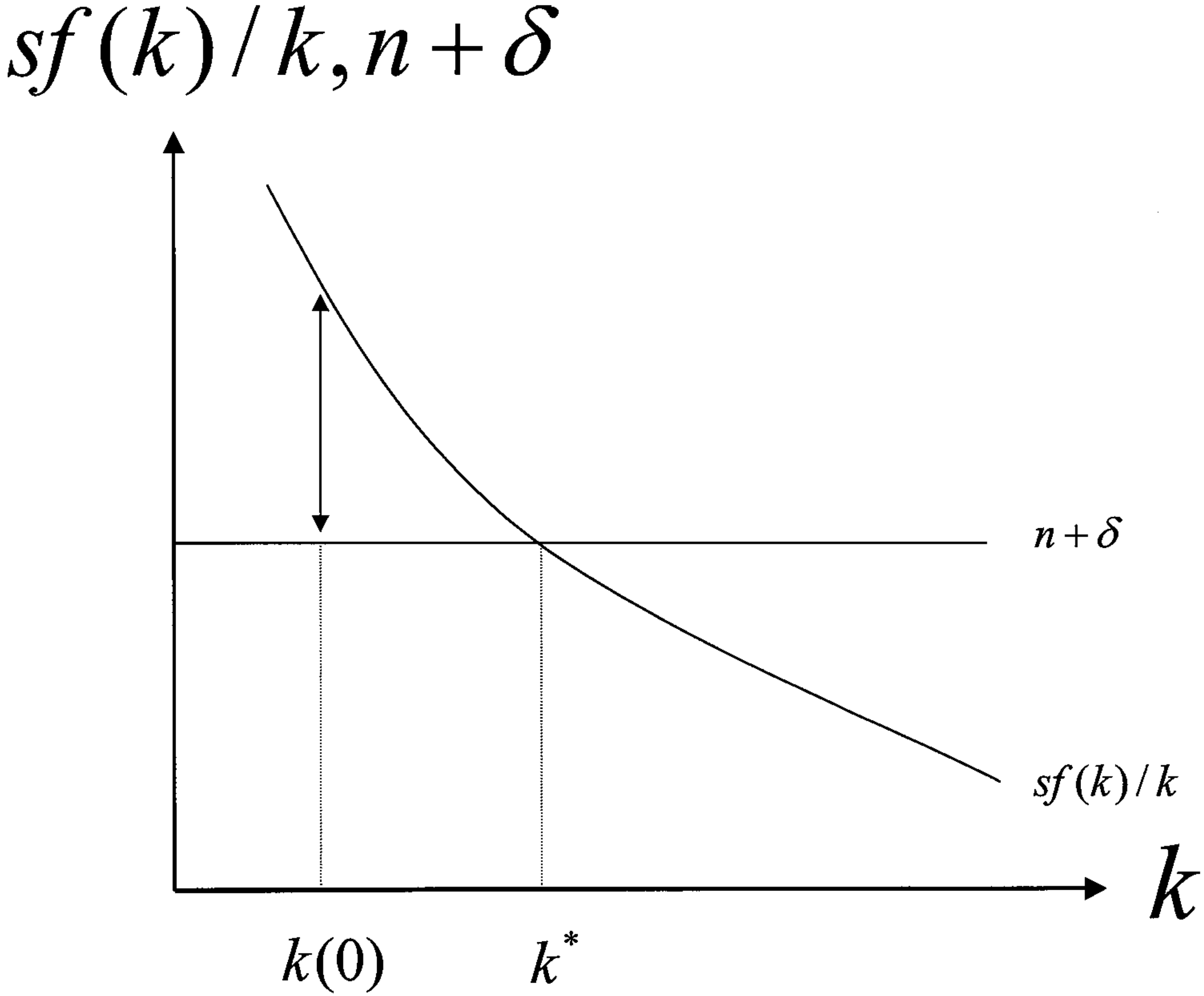

Fig. 2.

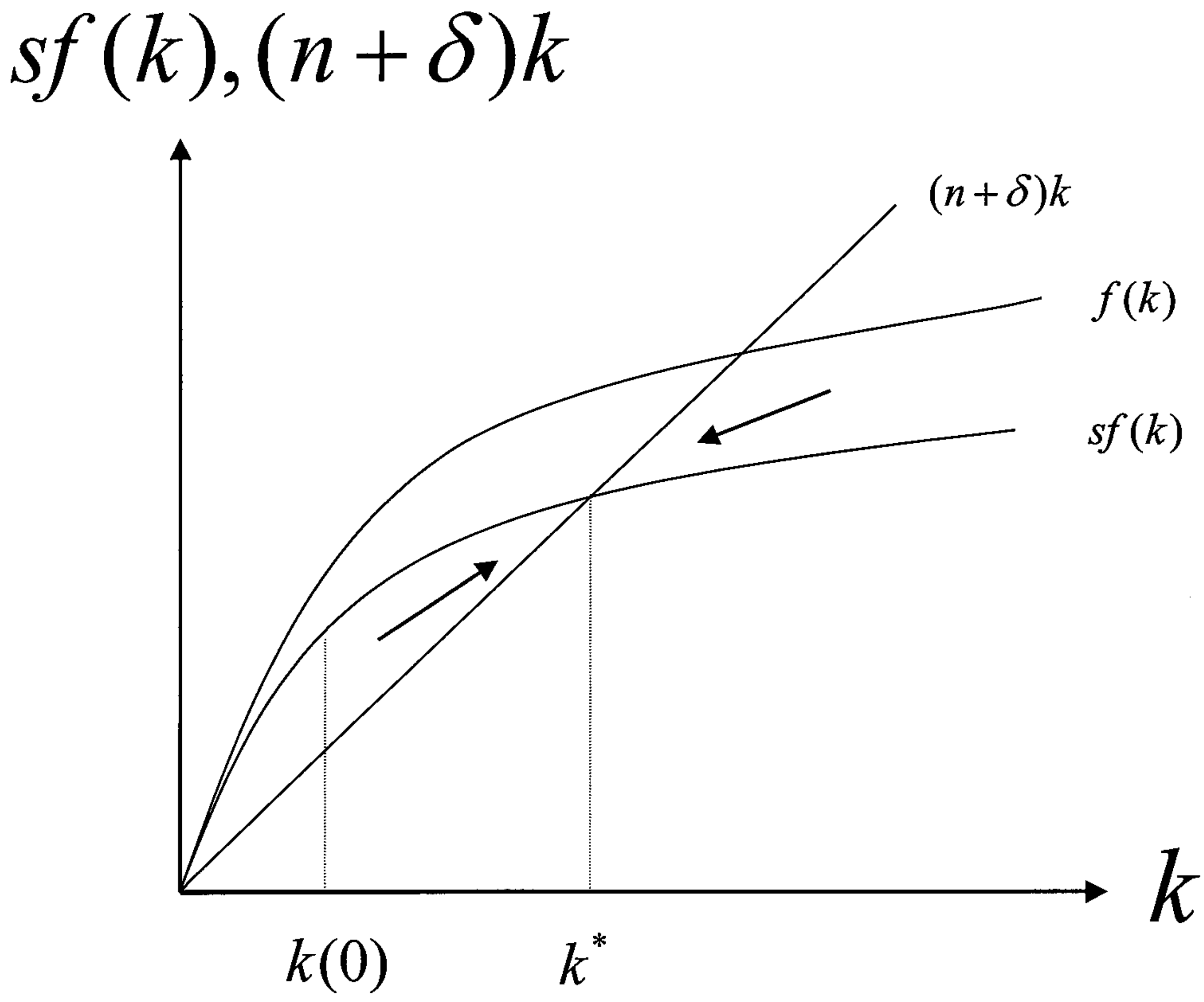

Fig. 1

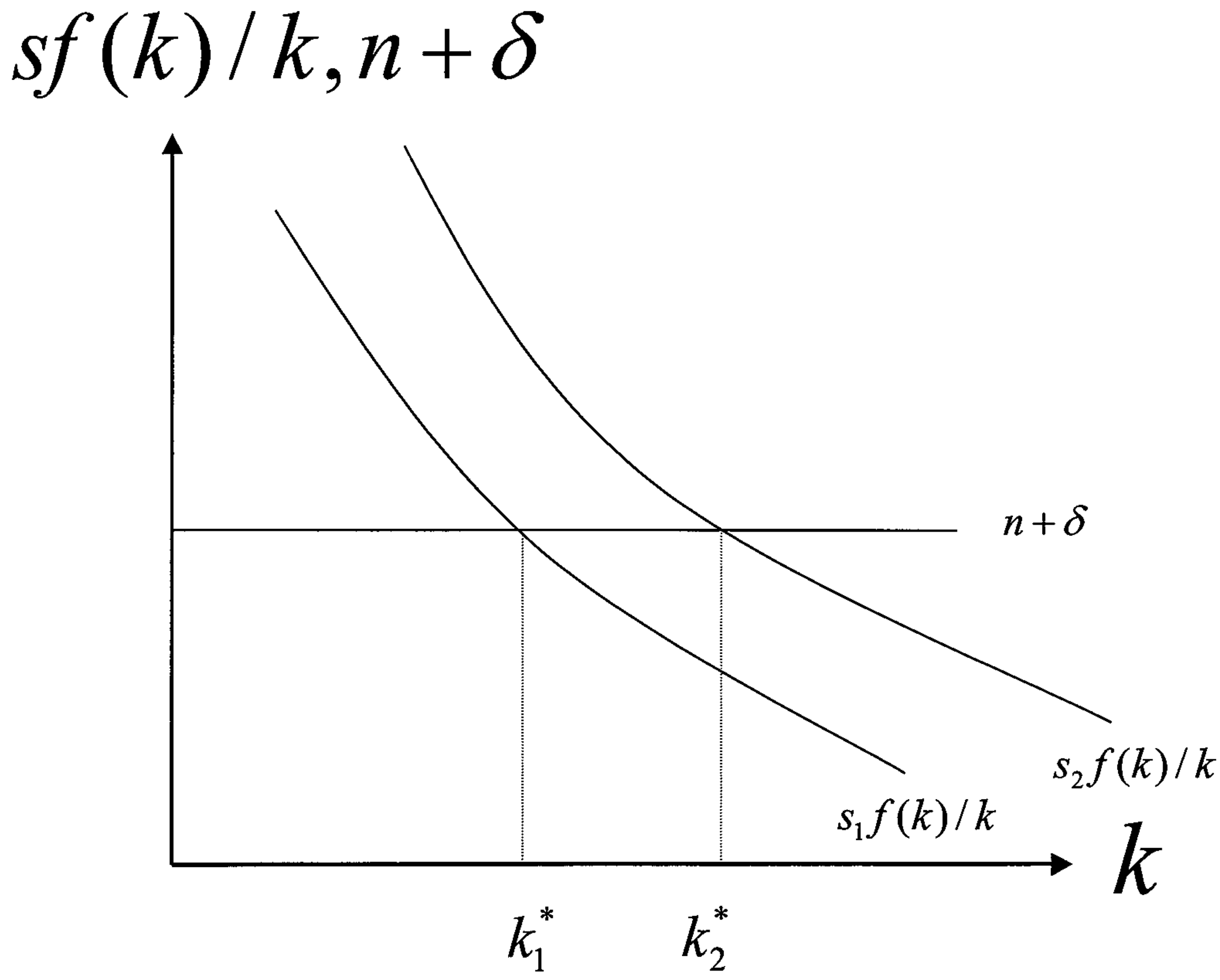

Fig. 3